INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

End Semester Examinations December 2018

B. Tech (Marine Engineering)

Semester – I

MATHEMATICS-I (UG11T2102)

Date:29-12-2018	Maximum Marks: 100
Time: 3Hrs	Pass Marks: 50

Note: i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part –A **Marks: 10 x 3 = 30**

(All Questions are compulsory)

1. a. State Leibnitz's theorem and find the n^{th} derivative of $y = e^x(2x+3)^3$

b. What is the formula for radius of curvature and centre of curvature at any point $P(x, y)$ on the curve $y = f(x)$

c. Find the equations of the tangent plane and the normal to the surface $2x^2 + y^2 + 2z - 3$ at $(2.2, -3)$

d. Evaluate by $\int_0^1 \frac{x^n - 1}{\log x}\, dx \;\; \alpha \geq 0$ differentiation under the integral sign.

e. What is the area of the region in the first quadrant bounded by the x axis and the curve $= \sin x$.

f. Find the volume of a sphere of radius a, using integration.

g. If $u = x^2 + y^2 + z^2$ and $\bar{V} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$, show that $div\,(u\bar{V}) = 5u$.

h. Given $\bar{A} = 2\bar{\imath} + 2\bar{\jmath} - \bar{k}$, $\bar{B} = 6\bar{\imath} - 3\bar{\jmath} + 2\bar{k}$ find the unit vector perpendicular to both $\bar{A}$ and $\bar{B}$.

i. Find the Eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

j. Evaluate by Cauchy's integral formula $\int_C \frac{z^2 - z + 1}{z - 1}\, dz$ where C is the circle $|z| = 1$.

Part – B Marks : 5 x 14 = 70

(Answer any 5 of the following 7 Questions)

(Note for Question Paper setters if necessary)

2. a. If $y = sin\,\log(x^2 + 2x + 1)$ prove that

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2 + 4)y_n = 0$$

b. Find the radius of curvature for the rectangular hyperbola $xy = c^2$.

(7+7 = 14)

3. a. The sum of 3 number is constant. Prove by Lagrange multiplier method their product is maximum when they are equal.

b. If $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$

show that $\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} + \frac{z\partial u}{\partial z} = 0$

(7+7 = 14)

4. a. Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $-2y + 8 = 0$.

b. Prove that $\beta(m.n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(7+7 = 14)

5. a. Evaluate $\int\int xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$.

b. Prove $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2}\log(1+a)$ where $a \geq 0$ by differentiation under the integral sign.

(7+7 = 14)

6. a. Using the line integral, commute the work done by the force $f = (2y+3)\bar{\imath} + xz\,\bar{\jmath} + (yz - x)\bar{k}$ when it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the curve $= 2t^2,\ \ y = t,\ \ z = t^3$.

b. If $\bar{A}$ and $\bar{B}$ are irrational, prove that $\bar{A} \times \bar{B}$ is solenoidal. (7+7 = 14)

7. a. Using Cayley Hamilton theorem, find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

b. Using Cayley Hamilton theorem that find the inverse of $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

(7+7 = 14)

8. a. Use Cauchy's integral formula to calculate $\int_C \frac{3z^2 + 7z+1}{z+1} dz$ where C is $|z| = \frac{1}{2}$.

b. Evaluate $\int_C \frac{z-3}{z^2 + 2z+5} dz$ where C is the circle $|z+1-i| = 2$ by using Cauchy's residue theorem.

(7+7 = 14)

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